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## CSET Mathematics: Properties of Vector Operations

### Addition and Scalar Multiplication

1.  $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2.  $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
3.  $\vec{a} + \vec{0} = \vec{a}$
4.  $\vec{a} + (-\vec{a}) = \vec{0}$
5.  $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$
6.  $(c + d)\vec{a} = c\vec{a} + d\vec{a}$
7.  $(cd)\vec{a} = c(d\vec{a})$
8.  $1\vec{a} = \vec{a}$

## Dot Product

The dot product is defined by

$$\vec{a} = \langle a_1, a_2, a_3 \rangle, \vec{b} = \langle b_1, b_2, b_3 \rangle$$
$$\implies \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

and obeys

0.  $\vec{a}, \vec{b}$  are vectors and  $\vec{a} \cdot \vec{b}$  is a number
1.  $\vec{a} \cdot \vec{a} = |\vec{a}|^2$
2.  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
3.  $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
4.  $(c\vec{a}) \cdot \vec{b} = c(\vec{a} \cdot \vec{b})$
5.  $\vec{0} \cdot \vec{a} = 0$
6.  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$
7.  $\vec{a} \cdot \vec{b} = 0 \iff \vec{a} = \vec{0} \text{ or } \vec{b} = \vec{0} \text{ or } \vec{a} \perp \vec{b}$

In property 6,  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ .

## Cross Product

The cross product is defined by

$$\vec{a} = \langle a_1, a_2, a_3 \rangle, \vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$\implies \vec{a} \times \vec{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$$

and obeys

0.  $\vec{a}, \vec{b}$  and  $\vec{a} \times \vec{b}$  are all vectors in three dimensions

1.  $\vec{a} \times \vec{b} \perp \vec{a}, \vec{b}$

2.  $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$

3.  $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$

4.  $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$

5.  $\vec{a} \times \vec{b} = 0 \iff \vec{a} = \vec{0} \text{ or } \vec{b} = \vec{0} \text{ or } \vec{a} \parallel \vec{b}$

6.  $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

7.  $(c\vec{a}) \times \vec{b} = \vec{a} \times (c\vec{b}) = c(\vec{a} \times \vec{b})$

8.  $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

9.  $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$

10.  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{c} \cdot \vec{a})\vec{b} - (\vec{b} \cdot \vec{a})\vec{c}$

In properties 2 and 4,  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ .

In property 4,  $|\hat{n}| = 1$ ,  $\hat{n} \perp \vec{a}, \vec{b}$  and  $(\vec{a}, \vec{b}, \hat{n})$  obey the right hand rule.

WARNING: Take particular care with properties 6 and 10. They are counterintuitive and cause huge numbers of errors. In particular,

$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$
$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

for most  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$ . For example

$$\hat{i} \times (\hat{i} \times \hat{j}) = \hat{i} \times \hat{k} = -\hat{k} \times \hat{i} = -\hat{j}$$
$$(\hat{i} \times \hat{i}) \times \hat{j} = \vec{0} \times \hat{j} = \vec{0}$$