Algebraic Expressions with Exponents on the ISEE

Upper Level

LESSON GOAL: Learn to multiply and divide algebraic expressions with exponents on the ISEE.

ISEE Question:

Simplify the expression: $\frac{a^{-7}\sqrt{a^4}}{a^8\,a^{-3}}$ A) a B) $\frac{1}{a^{10}}$ C) $a^{\frac{1}{10}}$ D) a^{10}

Normally, for a question with variables on the ISEE, the fastest solution is to plug in a number. Since we're working with big exponents, however, the only reasonable number to plug in is 1 (even 2 would create extremely big results). 1 raised to any power is always 1, so the entire expression would equal 1, but unfortunately, so would the answer choices. This is a rare instance where plugging in doesn't work.

Let's review the basic rules for exponents:

1. *When **multiplying** the same base, **add** the exponents:* $n^x n^y = n^{(x+y)}$, for example: $3^7 \times 3^2 = 3^9$
2. *When **dividing** the same base, **subtract** the exponents:* $\frac{n^x}{n^y} = n^{(x-y)}$, for example: $3^7 \div 3^2 = 3^5$
3. *A **square root** is the same as an **exponent of 1/2**:* $\sqrt{n} = n^{½}$
4. *A **negative exponent reciprocates the number**:* $n^{-x} = \frac{1}{n^x}$

Solution:

STEP 1: Simplify the square root (rule #3 above). $\sqrt{a^4} = a^{\frac{4}{2}} = a^2$

STEP 2: Flip the variables with negative exponents from numerator to denominator and vice versa to make them positive (rule #4). $\frac{a^{-7}\,a^2}{a^8\,a^{-3}} = \frac{a^3\,a^2}{a^8\,a^7}$

STEP 3: Add the exponents in the numerator, then in the denominator (rule #1). $\frac{a^3\,a^2}{a^8\,a^7} = \frac{a^5}{a^{15}}$

STEP 4: Simplify the fraction by subtracting the exponents (rule #2). $\frac{a^5}{a^{15}} = a^{-10}$

a^{-10} is not in the answer choices, but according to rule #4, we can reciprocate it to answer choice C) $a^{\frac{1}{10}}$

Helpful Tip: All this being said, this problem is both challenging and time-consuming, so it's a good one to skip, at least until you have given all other problems on this ISEE section a fair shot.